Teaching and learning mathematics at primary school

School resources

Mathematics is important in two ways, because:

1. it contains important skills and knowledge for participating in society as an adult, and
2. it serves as a gatekeeper for all sorts of opportunities in study and employment.

Young people need mathematics in order to live better lives[1]. Primary school mathematics is the foundation of all later mathematics learning, again in two ways:

1. the concepts of early mathematics are the roots of later ideas, and
2. young people's belief in their ability to do mathematics and their willingness to engage with mathematics are shaped by their primary school mathematics learning experiences.

How should I choose what to do when I teach mathematics?

How we teach mathematics in primary schools matters a lot. Every day, primary school teachers have to choose what mathematics they will teach, and how they will teach it. Each choice is significant, shaping what mathematics students will engage with, how students will approach mathematics, and the feelings that students develop about themselves and mathematics. All these factors make planning for, enacting and evaluating mathematics teaching and learning time-consuming and challenging for teachers.

Historically, and in other jurisdictions and levels of schooling, this problem has been addressed by trying to provide a comprehensive resource for teachers, such as a textbook or step-by-step handbook, that 'teacherproofs' the decision-making process by telling teachers what to do every step of the way. This often has the effect of 'learnerproofing' the curriculum too, with the pace of teaching being the same for everyone, and teachers 'delivering' what is needed in order to complete activities, with little participation from students. A contrasting approach is when teachers are positioned as adaptive experts whose job is to make evidence-informed choices about what students need, and then provide for them in a way that makes the students interact deeply with mathematical ideas. Teaching this way may be worthwhile, but it is not quick and easy – and it contains many challenges for primary school teachers. Teachers faced with these challenges are often seeking a magic bullet solution: a book, internet resource, curriculum or method that can support their choices and simplify the task of preparing mathematics lessons every day of the school year. As part of the (very reasonable) quest for manageability, the complexities of both mathematics and students can be sidelined.

Mathematical complexities

The first complexity that arises in teaching mathematics is **defining what we mean by mathematics**. This may seem like an intellectual game, but actually it is fundamentally important to what we teach and how to teach it. For example, if we define mathematics as a set of formal rules and procedures that must be mastered so that they can be performed automatically, then we would teach certain things in a certain way. If we define mathematics as a creative endeavour, we would teach different things in a different way. Many of the debates about teaching mathematics stem from this source. They represent disagreements about what mathematics should be learned and what successful mathematics learning looks like for primary-age children. Dichotomies of ‘basics versus problem solving’ or ‘mastery versus creativity’ can make it easy to talk about some of the differences, but they can also work against effective teaching. ‘Either/or’ thinking in education ignores context and professional decision making, both of which are central to effective teaching and learning.

The uncertainty with definitions is compounded by the nature of the mathematics that primary-age children are expected to learn. Mathematics is an established body of knowledge and skills that has been developed over millenia. Some believe this means that there is no room for discovery or creativity, that ‘reinventing the wheel’ wastes precious time and is foolish considering the standard nature of mathematics. Others think that children should explore the mathematics for themselves and rediscover the patterns and principles that underly the body of knowledge, because then they will make it ‘theirs’ and feel its importance and relevance. Distinctions are drawn between concepts and procedures, between knowledge and strategy, between facts and processes. Again, these dichotomies are ways to make plain the range of ways of thinking about mathematics teaching and learning, but they are rarely useful for effective teaching and learning. Teachers have to find the way between these signposts that best serves their students.

The final complexity arises from the fact that **some mathematics is hard to teach and tricky to learn**. Whilst mathematics is for everyone, and everyone can learn mathematics, teachers and students may still struggle with concepts along the way. There are key points where students and teachers often get ‘stuck’, and these key points have a significant impact on later mathematics: place value, multiplicative thinking, fractions, decimals, ratios and proportions, and algebra. In choosing what to do when teaching mathematics, it is necessary to understand these concepts, the best ways to represent them, and the best ways to help students understand them.

Student complexities

Students are not a uniform group, as teachers well know. They vary in many ways, and whenever they are in a learning situation they bring their full, complex selves to the task. When it comes to mathematics learning, teachers’ thinking about students is influenced by several key ideas that have permeated mathematics teaching over many decades. The ideas are so commonplace that we often do not recognise them, nor recognise their implications.

The first of these is **‘readiness’**. Teachers have first hand experience of this. Sometimes we try and try to work on an idea with a child, and we just can’t seem to make headway. Later the penny drops and suddenly they get it. What happened? Teachers often say they became ‘ready’ to learn. This idea derives from Piaget’s ideas about learning progressing in clearly defined stages. Piaget’s work has been hugely influential in mathematics teaching because it focused on ideas that underpin mathematics (like whether processes can be reversed, and whether the number of things in a group has changed) and it offered a way to understand the long plateaus and the sudden jumps teachers saw in children’s mathematics learning. The problem with the idea of ‘readiness’ is that it can stop teachers moving forward with students. If they don’t seem to be ‘ready’ for something we might stop too long and wait.

The second pervasive idea is mathematics **‘ability’**. Listen to any group of teachers chatting about their class’s mathematics learning and this idea will soon surface; within-class groupings are often referred to as ‘ability’ groups. Teachers, and students too, can quickly identify who they think is ‘good at maths’ or ‘bright’ or ‘top’. As with the idea of readiness, the idea of ability persists because it seems so visible in our mathematics classrooms. Some students pick up what is being taught quickly, and are able to apply it and extend their knowledge readily. Others seem to struggle to understand the concept the teacher is trying to explain. The problematic part is how quick teachers are to ascribe these differences to ‘ability’ –

perceived as a fixed, internal thing that some children have and others do not, as opposed to something that is changeable. Perhaps because mathematics has traditionally been seen as hard to learn and is used as a gatekeeper, it is easy to fall in the trap of thinking 'they are just not good at mathematics' or 'she's just a maths whizz' to explain differences between students. Ability might be a ready explanation, but it is not very helpful for teaching in ways that help all students progress. If teachers think some people are more able at mathematics than others, then they may give them more advanced mathematics to do, and withold the more advanced mathematics from those they think are less able. This seems like a good way to meet students' needs, but over time it restricts students' opportunity to learn, perpetuating the problem we are trying to solve. Teachers run the very real risk of underestimating students and 'under-teaching' them. However, looking at differences in pace of learning or amount of learning as being to do with experience, engagement and opportunities means there is more room to move, and more teachers can do to help. Saying 'all children can learn mathematics' is easy, but sincerely believing it is hard. It goes against our own learning experiences in many cases. It seems to go against our experiences as a teacher. However, really believing it, or at least teaching as though it is true until you have some more evidence, will make a noticeable difference to students' outcomes.

The third idea that permeates mathematics teaching, almost invisibly, is the idea of '**learning trajectories**', often presented as frameworks, lists, steps, stages and stairs that describe students' progress from naive to sophisticated ideas about mathematics. This idea has its origins in Piaget too, and was built on in the 1980s and 1990s when a key focus of mathematics education research was students' ways of thinking. Interviewing large numbers of young students revealed consistent patterns in their thinking, and how it changed over time, especially in counting and addition and subtraction. Further exploration revealed characteristics of 'multiplicative' and 'proportional' thinking, and common misconception patterns with fractions and decimals in particular. This research demonstrated more clearly the types of thinking students bring to learning mathematics at school, and it gave fascinating insight into what goes on in children's minds when they think about mathematics. What it did not do was suggest how mathematics should be taught as a result of these insights.

One result of finding a 'trajectory' that children seem to progress along was the notion that all teachers have to do is support the natural unfolding of students' thinking. Another assumption was that this trajectory operated like a set path or set of stairs, and therefore, until one thing is mastered, there can be no other progress. These conclusions underpin the construction of our current mathematics curricula that are based on learning trajectories. A third assumption was that if a clear and useful trajectory for one aspect of mathematics exists (for example, the development of counting) then all elements of mathematics can be described by trajectories. This assumption led to the invention of sequences and steps and stages, applying similar 'ladders' for students to climb to all areas of mathematics (beyond numeracy). Like many ideas in education, the core idea of learning trajectories in mathematics is sound, both in terms of research (it has been repeatedly shown to be true in several domains) and practice (we know from experience that students do hold common misconceptions that can be hard to get past). But also like many ideas in education, stretching it too far leads to unintended consequences. Progress may not be as linear as trajectories suggest; and it may be possible, or indeed important, to begin on a new 'ladder' while still climbing a previous one (for example, beginning to understand grouping and multiplication while still mastering addtion and subtraction). In addition, sticking closely to trajectories may lead to missing powerful links amongst ideas that go 'across' ladders (for example, grouping in tens, which underpins all four operations and metric measurement).

These three ideas - readiness, ability and trajectories – are all psychological in nature. They are about students' thinking and are metaphors that are used in education to explain how the brain works when it learns mathematics. But cognition and psychology are not the only student complexities that impact primary mathematics teaching and learning. A range of social factors also impacts students' progress in mathematics.

Teachers are very familiar with the idea that engagement impacts whether or not learning occurs. Engagement can be seen as a psychological idea, but it is also about social factors. Engagement is in part determined by whether the students' task is appropriate, but also by who the student is, what they are bringing to the task, their attitudes towards mathematics (Hard? Boring? Not for people like me?), their beliefs about mathematics (Is it learnable? Is it worthwhile?) and how they see themselves as students and, particularly, as students of mathematics. An important idea that studies have shown to impact students' mathematics engagement and progress is **mathematics learner identity**. Mathematics learner identity describes the relationships students form with mathematics and how they see themselves as a learner of mathematics. The relationship develops through experiences of learning mathematics, and it changes over time and in different contexts. How teachers choose to present and assess mathematics in their classrooms contribute to students' developing mathematics identities. Forming positive relationships with mathematics in primary school is important for later success in mathematics learning and students' decisions to continue the subject later in secondary school. This does not mean teachers need to make mathematics easy, nor disguise mathematics so the students 'don't even know they are doing it'. What is means is that they need to deliberately help students build a view of themselves as capable of learning mathematics, even if it is hard; and to value learning mathematics, showing them the relevance it has in their lives. This has implications for what mathematics is taught and how it is taught.

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Recommended further reading

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. Journal of Mathematics Education, 2 (2), 147-164.

Clements, D., & Sarama, J. (2004). Learning trajectories in mathematics education. Mathematical Thinking and Learning, 6 (2), 81-89.

Goos, M., & Kaya, S. (2020). Understanding and promoting students' mathematical thinking: A review of research published in ESM. Educational Studies in Mathematics, 103, 7–25.

Kullberg, A., Kempe, U., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? ZDM Mathematics Education, 49, 559-569.

OECD (2016). Equations and inequalities: Making mathematics accessible to all. PISA, OECD Publishing, Paris.

OECD (2016). Is memorisation a good strategy for learning mathematics? Pisa in Focus, 61.

Sullivan, P. (2011). Teaching mathematics: Using research informed strategies. ACER: Melbourne.

Endnotes

1 OECD (2016). Equations and inequalities: Making mathematics accessible to all.
PISA, OECD Publishing, Paris.

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Fiona Ell

Fiona Ell is an Associate Professor in the Faculty of Education and Social Work at the University of Auckland. She began her career as a primary school teacher before moving into teacher education and research. Fiona's research is concerned with how people learn in complex social settings, focused on how they learn mathematics and how they learn to teach mathematics. Recently she was part of the Royal Society panel that provided input for the Mathematics and Statistics curriculum refresh process.

Lisa Darragh

Dr. Lisa Darragh is a Senior Lecturer at the University of Auckland where she has taught mathematics education to pre-service and in-service teachers since February 2018. Prior to this she spent three years living in Santiago, Chile doing post-doctorate study at the Universidad de Chile. She completed her PhD in 2014, looking at children's mathematics learner identity as they transition to secondary school. Lisa also spent ten years as a teacher of Year 7-8 students at Kowhai Intermediate and Birkdale Intermediate schools and in Canada.